2-1 Rates of Change and Limits

Learning Targets

- I can find the value of a limit by looking at the graph of a funcon.
- I can calculate limits algebraically.
- I can calculate one and two sided limits.
- I can determine when a limit exists.

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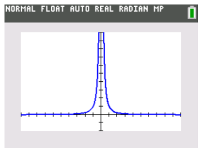
Graph $f(x) = \frac{1}{x^2}$



What happens to y as x gets closer to 0?

y is getting infinitely larger

What is happening at 0?

there is no y value

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Limits

Find $\lim_{x \to 0} \frac{1}{x^2} = \infty$

Find the limit of $\frac{1}{x^2}$ as x approachs 0

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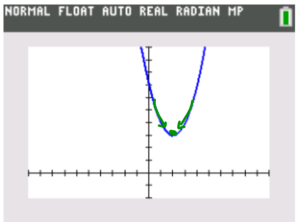
Find $\lim_{x \to 2} (x-2)^2 + 3$

$= 3$



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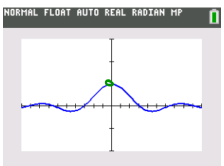
Find $\lim_{x \to 0} \frac{\sin x}{x} = 1$



Hole at 0

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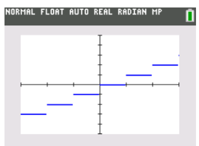
Find $\lim_{x \to 1} [\![x]\!]$

$=$ ONE

int (x)



$\lim_{x \to 1^+} [x] = 1$

$\lim_{x \to 1^-} [x] = 0$

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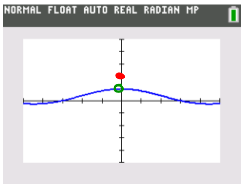
$$f(x) = \begin{cases} \frac{\sin x}{x} & if\ x \neq 0 \\ 2 & if\ x = 0 \end{cases}$$

Find $\lim_{x \to 0} f(x) = 1$



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Properes of Limits

$\lim_{x \to c} f(x) = L$ $\lim_{x \to c} g(x) = M$

- Sum Rule – $\lim_{x \to c} [f(x) + g(x)] = L + M$
- Difference Rule – $\lim_{x \to c} [f(x) - g(x)] = L - M$
- Product Rule – $\lim_{x \to c} [f(x) \cdot g(x)] = L \cdot M$

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Properes of Limits

4. Constant Mulple Rule - $\lim_{x \to c} [k \cdot f(x)] = k \cdot L$

5. Quoent Rule - $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}$

6. Power Rule - $\lim_{x \to c} (f(x))^k = L^k$

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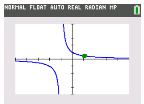
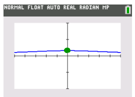
Ex1. Evaluate the limit (graphically)

1. $\lim_{x \to 1} \frac{x-1}{x^2-1}$

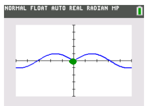


$\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} = \frac{1}{x+1} = \frac{1}{2}$

2. $\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2}$



3. $\lim_{x \to 0} \frac{x \sin x}{|x|}$



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Ex2. Evaluate the Limits (algebraically)

1. $\lim_{x \to 2} \frac{x^2 - 7x + 10}{x^2 - 4}$ $\frac{(x-5)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$\lim_{x \to 2} \frac{x-5}{x+2}$

$\frac{2-5}{2+2} = \frac{-3}{4}$

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2. $\lim_{x \to 0} \frac{(x+3)^3 - 27}{x} = \lim_{x \to 0} \frac{x^3 + 9x^2 + 27x}{x}$

$= \lim_{x \to 0} x^2 + 9x + 27$

$= 27$

$(x^2 + 6x + 9)(x+3)$

$x^3 + 3x^2 + 6x^2 + 18x + 9x + 27$

$x^3 + 9x^2 + 27x + 27$

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3. $\lim_{x \to 0} \frac{\tan x}{x}$

$= \lim_{x \to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \to 0} \frac{\sin x}{x \cos x}$

$= \lim_{x \to 0} \frac{1}{\cos x} = \frac{1}{\cos 0}$

$= 1$

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Ex3. Use the graph to evaluate

a. $\lim_{x \to 4} f(x)$ 2

b. $\lim_{x \to 2^-} f(x)$ 0

c. $\lim_{x \to 2^+} f(x)$ 2

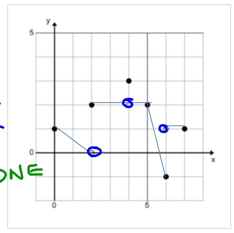
d. $\lim_{x \to 2} f(x)$ DNE

e. $\lim_{x \to 5} f(x)$ ~~5~~ 2

f. $\lim_{x \to 7} f(x)$ ~~1~~ DNE

g. $\lim_{x \to 7^-} f(x)$ 1

h. $\lim_{x \to 6} f(x)$ DNE



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Ex4. Find the limit.

$\lim_{x \to 0} \frac{x \sin x}{|x|}$

Sep 12-10:15 AM

Homework

p. 66 #7, 9, 11, 15, 16, 19, 20, 24, 27, 28, 31-36, 38, 40, 42, 44, 49

Sep 12-10:15 AM